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MAGNETIC MASS IN HOT SCALAR ELECTRODYNAMICS [1]

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Abstract

Using the Slavnov-Taylor identities we prove that the so-called "magnetic mass" is exactly equal to zero within hot scalar electrodynamics. The same result is valid for hot QED and seems for any abelian theory but this is not the case for hot QCD where one expects that $m^2_{mag} \neq 0$.

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At present for hot QCD and many other gauge theories it is very essential to calculate the so-called "magnetic mass," which is an infrared cutoff for gluomagnetic forces and in many cases it can protect this theory from infrared divergencies. This question has a very long history [1,2,3] but till now it is open to discussions. There are only the estimates made perturbatively for this parameter [4,5,6] although another possibility, which considers a nonanalytical behaviour [7,8], is also not excluded. Nevertheless this parameter (when $m^2_{mag} \neq 0$) is widely used today for many applications [9], especially when the next-to-leading order term is calculated [10] within hot QCD. Moreover it is often stated (starting from paper [11]) that for hot scalar electrodynamics and for any hot abelian theory this parameter is equal to zero although this fact has not been proven.

The goal of this paper is to calculate exactly the magnetic mass for hot scalar electrodynamics using the Slavnov-Taylor identities. Here we exploit the exact graph representation for the photon self-energy tensor and demostrate that, indeed, this parameter is equal to zero after the simple algebra being performed. Moreover we also see arguments that this result is valid for hot QED and it is correct for any abelian theory. For hot QCD $m^2_{mag} \neq 0$ although the analogous calculations are also valid. On the formal level, the graphs with other numerical coefficients define the QCD self-energy tensor but, of course, the real reason is connected with the essential different nature of hot QCD infrared divergencies.

Scalar electrodynamics is determined through the Lagrangian

$$\mathcal{L}_A = -\frac{1}{4}F^2_{\mu\nu} - |(\partial_\mu - ieA_\mu)\phi|^2 - \frac{\lambda}{4}(\phi^+\phi)^2 \tag{1}$$

where A_μ is an abelian gauge field and $\phi^+(\phi)$ are the complex scalar ones. Here $F_{\mu\nu}$ is the standard electromagnetic field strength tensor and the last term in Eq.(1) is necessary to make the model (1) renormalizable. The quantum Lagrangian for the theory under consideration is built as usual and has the form

$$\mathcal{L} = \mathcal{L}_A + \mathcal{L}_{g.f.}$$
$$\mathcal{L}_{g.f} = \frac{1}{2\alpha}(\partial_\mu A_\mu)^2 + \bar{C}(\partial^2_\mu)C \tag{2}$$

where we add terms which fix the gauge and the appropriate ghost fields.

The set of equations for the temperature Green functions can be easily obtained via the stationary-action principle [12] and has the standard Schwinger-Dyson form

$$D^{-1}(k_4, \mathbf{k}) = D_0^{-1}(k_4, \mathbf{k}) + \Pi(k_4, \mathbf{k})\,, \qquad G(k_4, \mathbf{k}) = G_0^{-1}(k_4, \mathbf{k}) + \Sigma(k_4, \mathbf{k}) \quad (3)$$

where Π and Σ are the self-energy part of the photon Green function and the Green function of scalar fields, respectively. The explicit form of Π can be represented by the four nonperturbative graphs

(4)

where all lines and the bold points should be identified with the exact Green and vertex functions. All the bare vertices are found to be

$$\begin{gathered} \Gamma^0_{A\phi\phi^+}(k|p+k,p)_\mu = e(2p+k)_\mu \\ \Gamma^0_{A^2\phi\phi^+}|_{\mu\nu} = -2e\delta_{\mu\nu}\,, \quad \Gamma^0_{(\phi\phi^+)^2} = -\lambda \end{gathered} \quad (5)$$

and they are independent from the gauge chosen. The last two functions are independent from momenta as well.

For the Feynman gauge (where $\alpha = 1$) the photon self-energy tensor is transversal

$$k_\mu \Pi_{\mu\nu}(k) = 0 \quad (6)$$

and can be represented with the aid of two scalar functions in the form

$$\begin{aligned} \Pi_{ij}(k_4, \mathbf{k}) &= (\delta_{ij} - \frac{k_i k_j}{\mathbf{k}^2}) A(k_4, \mathbf{k}) + \frac{k_i k_j}{\mathbf{k}^2}\frac{k_4^2}{\mathbf{k}^2}\Pi_{44}(k_4, \mathbf{k}) \\ \Pi_{i4}(k_4, \mathbf{k}) &= \Pi_{4i}(k_4, \mathbf{k}) = -\frac{k_i k_4}{\mathbf{k}^2}\Pi_{44}(k_4, \mathbf{k})\,, \quad i,j = 1,2,3 \end{aligned} \quad (7)$$

The magnetic mass is determined to be

$$m^2_{mag} = A(k_4 = 0, \mathbf{k} \to 0) \quad (8)$$

where the limit is defined in the infrared manner. However, since $\Pi_{44}(k_4 = 0, \mathbf{k} \to 0) \neq 0$ for this theory, it is more convenient to use for calculating m^2_{mag} the relation

$$m^2_{mag} = \frac{1}{2}\sum_i \Pi_{ii}(k_4 = 0, \mathbf{k} \to 0) \quad (9)$$

which directly follows from Eq.(7) when the Feynman gauge is used.

Our tool for transforming Eq.(4) is the exact Slavnov-Taylor identities

$$
\begin{aligned}
\Gamma_{A\phi\phi^+}(0|p,p)_i &= e\frac{\partial G^{-1}(p)}{\partial p_i} \\
\Gamma_{A^2\phi\phi^+}(0,k|p+k,p)_{ij} &= -e\frac{\partial \Gamma_{A\phi\phi^+}(k|p+k,p)_i}{\partial p_j}
\end{aligned}
\tag{10}
$$

which can be found by using the known prescription [12]. They are valid if one momentum is equal to zero in the infrared manner and for indices $i,j \neq 4$.

One-loop nonperturbative graphs and two-loop ones in Eq.(4) are canceled independently. The two first (one-loop) graphs are easily put in the form

$$
\Pi^{(1)}_{ii}(0) = \frac{6e^2}{\beta}\sum_{p_4}\int\frac{d^3p}{(2\pi)^3}G(p) - \frac{e^2}{\beta}\sum_{p_4}\int\frac{d^3p}{(2\pi)^3}(2p_i)G(p)\frac{\partial G^{-1}(p)}{\partial p_i}G(p) \tag{11}
$$

where we have used the first formula from Eq.(10) to eliminate the $\Gamma_{A\phi\phi^+}$-function. Then using that $GG^{-1}=1$ we arrive at the expression

$$
\Pi^{(1)}_{ii}(0) = \frac{6e^2}{\beta}\sum_{p_4}\int\frac{d^3p}{(2\pi)^3}G(p) + \frac{e^2}{\beta}\sum_{p_4}\int\frac{d^3p}{(2\pi)^3}(2p_i)\frac{\partial G(p)}{\partial p_i} \tag{12}
$$

which is exactly equal to zero when one calculates the last integral by parts (the surface term being zero is ommited [13]). On this level we have that the nonperturbative one-loop $\Pi^{(1)}_{ii}(0)=0$. The same situation with the one-loop nonperturbative graphs takes place also in hot QCD which is possible to see, for example, in axial temporal gauge [13]. For hot QED the analogous calculations prove that $m^2_{mag}=0$ at once since the exact graph representation for the photon self-energy part in QED does not contain the nonperturbative two-loop graphs [12].

But there is a problem when the nonperturbative two-loop graphs are considered. For the model (1), however, we demostrate that the two last nonperturbative graphs in Eq.(4) seem to be equal to zero as well. Here we take the third graph (below called G_3) from (4) which (after the first formula from Eq.(10) being used) has the form

$$
(G_3) = \frac{2e^3}{\beta^2}\sum_{k_4,p_4}\int\frac{d^3k}{(2\pi)^3}\frac{d^3p}{(2\pi)^3}D_{ij}(k)G(p+k)\Gamma_j(k|p+k,p)\frac{\partial G(p)}{\partial p_i} \tag{13}
$$

and we perform the intergation by parts within Eq.(13). If the integral (below K-term)

$$K = \frac{2e^3}{\beta^2} \sum_{k_4, p_4} \int \frac{d^3k}{(2\pi)^3} \frac{d^3p}{(2\pi)^3} D_{ij}(k) \left[\frac{\partial}{\partial p_i} G(p+k) \Gamma_j(k|p+k, p) G(p) \right] \tag{14}$$

is equal to zero, the expression Eq.(13) becomes

$$(G_3) = -\frac{2e^3}{\beta^2} \sum_{k_4, p_4} \int \frac{d^3k}{(2\pi)^3} \frac{d^3p}{(2\pi)^3} D_{ij}(k) \left[\frac{\partial}{\partial p_i} G(p+k) \Gamma_j(k|p+k, p) \right] G(p) \tag{15}$$

and this representation for G_3 is enough to prove within the model(1) that $m^2_{mag} = 0$ exactly. Now one should explicitly perform a differentiation within Eq.(15) and find the simple identity for the exact graphs within Eq.(4)

$$\tag{16}$$

which shows that the magnetic mass for this model is indeed equal to zero

$$m^2_{mag} = 0 \tag{17}$$

However we should prove else that from Eq.(14) $K = 0$. In the lowest perturbative order (here this means the e^4-term) one can demonstrate that $K^{(0)} = 0$, transforming Eq.(14) to the form

$$K^{(0)} = \frac{2e^4}{\beta^2} \sum_{k_4, p_4} \int \frac{d^3k}{(2\pi)^3} \frac{d^3p}{(2\pi)^3} \frac{1}{k^2} \left[\frac{6}{(p+k)^2 p^2} - \frac{8\mathbf{p}^2 + 4\mathbf{pk}}{(p+k)^2 p^4} \right] \tag{18}$$

and then calculates it in the usual manner. For example, using the infrared manner of calculation, one finds at once that

$$K^{(0)} = \frac{2e^4}{\beta^2} \int \frac{d^3k}{(2\pi)^3} \frac{d^3p}{(2\pi)^3} \frac{1}{\mathbf{k}^2} \left[-\frac{2}{(\mathbf{p}+\mathbf{k})^2 \mathbf{p}^2} - \frac{4\mathbf{pk}}{(\mathbf{p}+\mathbf{k})^2 \mathbf{p}^4} \right] = 0 \tag{19}$$

So there is not any problem with the leading g^4-term calculated for m^2_{mag} and it being zero strongly indicates that $m^2_{mag} = 0$ within all perturbative orders.

For hot QCD $m^2_{mag} \neq 0$ already within the g^4-order [4] although the analogous calculations are also possible, for example, in the axial temporal gauge. On the formal level the graphs with other numerical coefficients define the QCD self-energy tensor but, of course, the real reason is connected with the essential different nature of the QCD infrared divergencies.

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